A Variable Structure Approach to Robust Control of VTOL Aircraft

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This paper examines the application of variable structure control theory to the design of a flight control system for the AV-8A Harrier in a hover mode. The objective in variable structure design is to confine the state trajectories to a subspace of the total state space. The motion in this subspace is insensitive to system parameter variations and external disturbances that lie in the range space of the control. A switching type of control law results from the design procedure. The control system was designed to track a vector-valued velocity command. For comparison, a proportional controller was designed using optimal linear regulator theory. Both controllers were evaluated for their transient response performance using a linear model; then a nonlinear simulation study of a hovering approach to landing was conducted. The variable structure controller outperformed its linear counterpart in the presence of wind disturbances and plant parameter uncertainties afforded by the simulation.

Nomenclature

A	= plant matrix
B	= control matrix
$DU, UB, \delta u$	= velocity perturbation along body x axis, m/s
$DV, VB, \delta v$	= velocity perturbation along body y axis, m/s
K	= feedback gain matrix
LAT, δLAT	= lateral stick perturbation, cm
LONG, δLON	= longitudinal stick perturbation, cm
p	= body-axis roll rate, rad/s
q	= body-axis pitch rate, rad/s
Q	= state performance index weighting matrix
r	= body-axis yaw rate, rad/s
R	= control performance index weighting matrix
RG	= normal offset from the approach trajectory, m
RLATSTK	= lateral stick, cm
RLONSTK	= longitudinal stick, cm
RN	= engine speed perturbation, % rpm
RN_C	= throttle perturbation, deg
RUD, δRUD	= rudder pedal perturbation, cm
$S2$	= distance from lateral switching surface, rad/s
THTN	= engine nozzle angle perturbation, rad
THTNC	= engine nozzle angle command, deg
$u_c, u_{\rm com}$	= velocity command along body x axis, m/s
$v_c, v_{\rm com}$	= velocity command along body y axis, m/s
$w_c, w_{\rm com}$	= velocity command along body z axis, m/s
$\delta\psi$	= Euler yaw attitude perturbation, rad
θ	= Euler pitch angle perturbation, rad
ϕ	= Euler roll attitude perturbation, rad
ψ_c	= heading command, rad

I. Introduction

FEEDBACK control design techniques typically depend on having a linear time invariant state model description or transfer function representation of the open loop plant dynamics. Since all real plants are nonlinear, the linear representation is obtained by either ignoring nonlinearities, or by expanding the equations of motion to the first order in the states and control about a specified operating point. This is particularly true in the design of flight control systems. The linearized perturbation model is an approximation to the aircraft dynamics at a selected flight condition. If the controller is to operate over a spectrum of flight conditions within the flight envelope, then it is customary to regard the transitions from one flight condition to the next as giving rise to slow variations in the model parameters. This leads to the design of flight control systems with scheduled gains. If these transitions take place slowly, then it is hoped the stability and transient performance criteria used in the linear design are preserved in actual flight. In practice, this is found to be the case to varying degrees, depending on the aircraft dynamics and the maneuvers performed.

The viewpoint taken in this paper is that modelling errors due to nonlinearities can be represented as unknown but bounded variations in the parameters of the open loop, linear state model description of the plant dynamics. A nonlinear control is designed that meets transient performance specifications, and at the same time is nearly insensitive (robust) with respect to some of the uncertain parameters that belong to a specified class. The resulting design is referred to as a variable structure control, because the control gains (and thus the closed loop system dynamics) switch according to the state location with reference to a surface in the state space. The control is designed to cause the motion of the system to reach and stay in the neighborhood of this surface. In the case of m controls, motion is desired at the intersection of m defined surfaces. When the motion is in the so called "sliding mode" along the surface (or intersection of surfaces), the system dynamics are insensitive with respect to a class of parameter variations. More importantly, stability is guaranteed so long as the variations remain within the specified bounds.

II. Variable Structure Design

The design of aircraft flight control systems is often complicated by the presence of system parameter variations, nonlinearities, cross-coupling effects and external disturbances that are ignored in the initial design phases. Traditionally, these effects have been diminished through a combination of high gain and integral feedback. But because of the limited control power available in STOL and VTOL type aircraft, it is likely that such approaches lead to control saturation, even when the demand for trim control power is modest. The

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Variable Structure System (VSS) design approach investigated here is nearly insensitive to a class of system parameter variations and provides good disturbance rejection properties without the use of high gain.

A VSS control law is a discontinuous feedback law whose feedback gain coefficients switch on hypersurfaces defined in the state space. The important feature of VSS is the "sliding mode," which may occur on the intersection of the switching surfaces. While in the sliding mode, the feedback system is insensitive to certain parameter variations and disturbances. The theory of VSS has been mainly developed in the U.S.S.R.,[1,2] and a survey can be found in Ref. 3.

To illustrate the parameter invariance and disturbance rejection properties, consider a controllable linear system in the following form:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2 \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + Bu + g(t) \end{aligned} \tag{1}$$

where x_1 is $(n-m)$-dimensional; x_2, $g(t)$, and u are m-dimensional, and B is invertible. The matrices A_{21}, A_{22}, and B possess elements that are unknown but bounded. In general, they are time varying due to the presence of nonlinearities in the original system. The vector $g(t)$ represents disturbances and unmodelled dynamics, including coupling effects from other subsystems.

In variable structure control, u is chosen so that trajectories are attained near the intersection of m hypersurfaces. The most acceptable surfaces for design purposes are stationary hyperplanes

$$s = C_1 x_1 + x_2 = 0 \tag{2}$$

where C_1 is an $m \times (n-m)$ matrix to be chosen as part of the design process. When the trajectory is in the vicinity of Eq. (2), it is said to be in the sliding mode. Using Eq. (2) in the first equation of Eq. (1) reduces the dynamics to

$$\dot{x}_1 = (A_{11} - A_{12}C_1)x_1 \tag{3}$$

Thus, the motion in the sliding mode is invariant with respect to system parameter variations and external disturbances. In order to reach and maintain sliding, a control is designed from a class of switching controls:

$$u = \begin{cases} u_i^+(x), & s_i > 0 \\ u_i^-(x), & s_i < 0 \end{cases} \tag{4}$$

where s_i is the ith element in s. Several good numerical examples of the robustness properties of variable structure control are given in Ref. 4.

The design process consists of two steps. First, select C_1 so that the system has desirable properties in the sliding mode, then choose u_i^+ and u_i^- to guarantee reaching and existence of the sliding mode over the feasible part of the state space. The first step is an ordinary problem in control theory since it relates to the placement of the eigenvalues of the reduced system, Eq. (3). An approach proposed in Ref. 5 is to regard x_2 as a control variable in the first equation of Eq. (1) that minimizes a quadratic performance of the form

$$J = \frac{1}{2}\int_{t_1}^{\infty} \left(x_1^T Q x_1 + x_2^T R x_2\right) \mathrm{d}t \tag{5}$$

where t_1 is defined as the time when sliding motion is initiated. In this case

$$C_1 = R^{-1}A_{12}^T P \tag{6}$$

where P satisfies

$$PA_{11} + A_{11}^T P - PA_{12}R^{-1}A_{12}^T P + Q = 0, \qquad P > 0 \tag{7}$$

Step 2 of the design process is fairly involved for the case of vector control,[1] particularly in the presence of external disturbances where it is necessary to either estimate the disturbance[6] or use a servo-compensator.[7] However, in the case of scalar control, and in the absence of external disturbances, reaching and existence of a sliding mode are guaranteed when $s\dot{s} < 0$. This leads to the following control structure

$$u = -\sum_{i=1}^{n} \psi_i x_i \tag{8}$$

The gains ψ_i are switched according to

$$\psi_i = \begin{cases} \alpha_i, & x_i s > 0 \\ \beta_i, & x_i s < 0 \end{cases} \tag{9}$$

where α_i and β_i satisfy the following inequalities:

$$\mathrm{sign}(B)\alpha_i \geq \sup_t \left[ca^i/|B|\right] \tag{10}$$

$$\mathrm{sign}(B)\beta_i \leq \inf_t \left[ca^i/|B|\right] \tag{11}$$

In Eqs. (10) and (11), B is now a scalar, C is a row vector

$$C = [c_1, 1] \tag{12}$$

and a^i is the ith column in the composite system matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{13}$$

Note that the spread between α_i and β_i increases with the uncertainty in A_{21}, A_{22}, and B. This results in larger jumps in u when s changes sign. Since the control tends to chatter when the system is in the sliding mode, this becomes a major design consideration. On the other hand, due to the product form in Eq. (8), the switching level decreases in proportion to x_i, and approaches zero as the system is driven to the origin.

III. Application to AV-8A in Hover

This section summarizes a VSS design that results in a velocity command control system for the AV-8A Harrier dynamics in hovering flight. The linearized dynamic model for the Harrier was taken from Ref. 8 for a trim relative wind speed of 15.43 m/s (30 knots). In addition, a linear regulator design is given for the same flight condition. This design was done so that the closed loop eigenvalues closely matched the dynamics of the VSS design in the sliding mode. The resulting linear regulator was used as a baseline for comparing the performance of the VSS design.

VSS Design for Longitudinal Dynamics

Reference 8 gives the linearized model for the AV-8A Harrier in hover for airspeeds between 0 and 120 knots. Selecting the values for 30 knots, we have the following model for the longitudinal system dynamics in the body frame:

$$\dot{x} = Ax + Bu \tag{14}$$

where

$$x^T = [u, w, \theta, q, RN] \tag{15}$$

$$u^T = [\delta\text{LON}, \text{THTN}, RN_c] \tag{16}$$

$$A = \begin{bmatrix} -0.035 & -0.02 & -9.8 & 0 & 0.002 \\ -0.011 & -0.105 & 0 & 0 & -0.309 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0056 & 0.014 & 0 & -0.13 & 0.0016 \\ 0 & 0 & 0 & 0 & -4.86 \end{bmatrix} \tag{17}$$

$$B = \begin{bmatrix} 0 & -9.8 & 0 \\ 0.063 & 0.28 & 0 \\ 0 & 0 & 0 \\ 0.079 & 0 & 0 \\ 0 & 0 & 4.86 \end{bmatrix} \tag{18}$$

The engine response time constant (1/4.86 s) was taken from Table 14 of Ref. 9 for an $RN = 86.5\%$.

The VSS design for surge velocity control was based on the controller structure shown in Fig. 1. Since the VS control in this structure is used for attitude stabilization, the sliding surface is defined in terms of θ and q:

$$s = C_1 \delta\theta + q, \qquad \delta\theta = \theta - \theta_c \tag{19}$$

where

$$\theta_c = k_1 \delta u, \qquad \delta u = u - u_c \tag{20}$$

and u_c is regarded as a constant or slowly varying input. In the sliding mode ($s = 0$) we have from Eqs. (17) and (19)

$$\tau_1 \dot{\theta} = -\theta + \theta_c, \qquad \tau_1 = 1/C_1 \tag{21}$$

which is stable from any $C_1 > 0$. The transient response is dictated by C_1 and is invariant with respect to remaining state variables. The design of k_1 and C_1 is based on Fig. 2. The closed loop poles were chosen to give a natural frequency of 1.0 rad/s and a damping ratio of 0.70. The resulting values for C_1 and k_1 are:

$$C_1 = 1.4 \text{ s}^{-1}; \qquad k_1 = 0.0729 \text{ s/m} \tag{22}$$

The heaving motion is controlled using a conventional proportional control law that results in a natural frequency of 3.5 rad/s and a damping of 0.70.

$$RN_c = 8.04 \delta w \tag{23}$$

$$\delta w = w - w_c \tag{24}$$

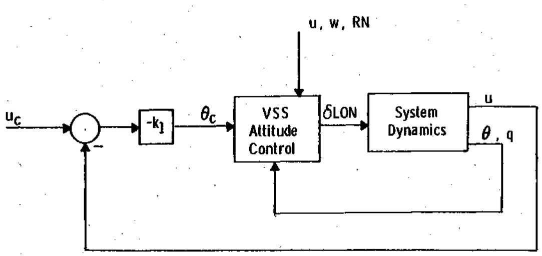

Fig. 1 Controller structure for surge dynamics.

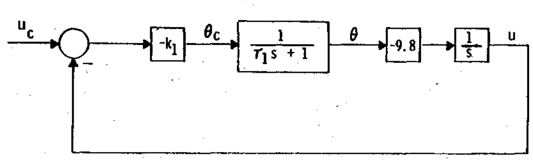

Fig. 2 Approximate model for surge dynamics in sliding mode.

This is accounted for by modifying $A_{5,2}$ in Eq. (17) from 0.0 to 39.1 and deleting the last column in Eq. (18).

To guarantee reaching and existence of the sliding mode, it is sufficient that

$$s\dot{s} < 0 \tag{25}$$

Differentiating Eq. (19) and assuming $\dot{u}_c = 0$, we obtain

$$s\dot{s} = s\left[\sum_{i=1}^{5} d_i x_i + 0.079 \delta\text{LON} + 9.8 C_1 k_1 \text{THTN}\right] \tag{26}$$

where the nominal values for d_i are:

$$\bar{d}_1 = 0.0056 + C_1 k_1 (0.035), \qquad \bar{d}_4 = C_1 - 0.13$$

$$\bar{d}_2 = 0.014 + C_1 k_1 (0.02), \qquad \bar{d}_5 = 0.0016 - C_1 k_1 (0.002)$$

$$\bar{d}_3 = C_1 k_1 (9.8) \tag{27}$$

To satisfy Eq. (25), the following control structure is used

$$\delta\text{LON} = -\sum_{i=1}^{5} \psi_i x_i; \qquad \psi_i = \begin{cases} \alpha_i, & s x_i > 0 \\ \beta_i, & s x_i < 0 \end{cases} \tag{28}$$

$$\text{THTN} = -6.0 s \tag{29}$$

where

$$\alpha_i > \max\{\bar{d}_i / 0.079\}, \qquad \beta_i < \min\{\bar{d}_i / 0.079\} \tag{30}$$

Allowing for possible variations of 100% in the parameters in Eq. (17), with the exception of $A_{1,3}$ the following selections were made:

$$\alpha = \begin{bmatrix} 0.232 \\ 0.406 \\ 13.9 \\ 19.5 \\ 0.041 \end{bmatrix} \qquad \beta = \begin{bmatrix} 0 \\ 0 \\ 11.4 \\ 16.0 \\ 0.005 \end{bmatrix} \tag{31}$$

The nozzle angle control in Eq. (29) was used to control the rate in reaching the sliding surface, and does not affect the dynamics in the sliding mode. In general, it was found that while the VS control in Eq. (28) is sufficient to guarantee reaching of the sliding surface, the time duration of the reaching phase can be large in the absence of a term proportional to s. In the lateral dynamics, this term was added directly to δLAT, since direct side force control is not available in the Harrier.

VSS Design for Lateral Dynamics

From the data in Ref. 8 at 30 knots, the lateral dynamics are defined by:

$$\dot{x} = Ax + Bu \tag{32}$$

Table 1 Eigenvalues and eigenvectors for lateral dynamics in sliding mode

State variables	Eigenvalues $-0.727 \pm i0.683$	$-1.18 \pm i0.908$
$\delta\psi \times 10^2$	$0.077 \pm i1.14$	$12.4 \pm i11.4$
$\phi \times 10^2$	$-6.4 \pm i6.74$	$-1.15 \pm i2.60$
δv	1.0	1.0
$r \times 10^2$	$-0.84 \mp i0.78$	$-2.50 \mp i2.23$

where

$$x^T = [\psi, \phi, v, r, p] \tag{33}$$

$$u^T = [\delta\text{LAT}, \delta\text{RUD}] \tag{34}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 9.8 & -0.042 & 0 & 0 \\ 0 & 0 & -0.007 & -0.06 & -0.075 \\ 0 & 0 & -0.039 & 0.11 & -0.260 \end{bmatrix} \tag{35}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -0.27 \\ 0.0055 & 0.085 \\ 0.177 & -0.033 \end{bmatrix} \tag{36}$$

In this case, it was decided to first determine δRUD as a conventional control to maintain a desired heading for the stability axis frame. Ignoring $A_{4,3}$ and $A_{4,5}$ in Eq. (35), and for a natural frequency and damping of 1.5 rad/s and 0.8, respectively, we obtain

$$\delta\text{RUD} = -26.5\delta\psi - 28.2r \tag{37}$$

where

$$\delta\psi = \psi - \psi_c \tag{38}$$

Substituting Eq. (37) into Eq. (32) results in the following single input system

$$\dot{x} = Ax + b\delta\text{LAT} \tag{39}$$

where

$$x^T = [\delta\psi, \phi, \delta v, r, p] \tag{40}$$

$$A = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 7.21 & 9.8 & -0.042 & 7.67 & 0 \\ -2.25 & 0 & -0.007 & -2.46 & -0.075 \\ \hline 0.875 & 0 & -0.039 & 1.04 & -0.26 \end{array}\right] \tag{41}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0055 \\ \hline 0.177 \end{bmatrix} \tag{42}$$

$$\delta v = v - v_c \tag{43}$$

Note that δRUD has a significant affect on lateral velocity. This can be seen in comparing Eq. (41) with Eq. (35).

In accordance with the design procedure outlined in Sec. II, the coefficients of the sliding surface were obtained by regarding p as a control variable for the reduced system:

$$\dot{x}_I = A_I x_I + b_I p \tag{44}$$

where A_I and b_I are the upper left and right blocks of A in Eq. (41). A sliding surface

$$p = -c_I x_I = -R^{-1} b_I^T P x_I \tag{45}$$

was determined that minimizes a quadratic performance index of the form in Eq. (5). It was found after several iterations that the following choice of Q and R resulted in a suitable set of closed loop eigenvalues (Table 1):

$$Q = \text{diag}[1, 1, 0.1, 1], \qquad R = 10 \tag{46}$$

A natural frequency of 1.0 rad/s with adequate damping appears to be a suitable design criterion for both longitudinal and lateral velocity control for attitude only systems, which is the case here in the sliding mode. While this has not been validated completely, the results in Ref. 10 indicate a pilot preference for sluggish response in deference to large attitude excursions.

The above design resulted in a sliding surface

$$s = Cx, \qquad C = [0.104, 1.4, 0.09, 0.312, 1.0] \tag{47}$$

Once again, to guarantee reaching and existence of the sliding mode, we must insure that $s\dot{s} < 0$. Differentiating Eq. (47) and assuming that $\dot{v}_c = \dot{\psi}_c = 0$, we have

$$s\dot{s} = s\left[\sum_{i=1}^{5} d_i x_i + d_6 \delta\text{LAT}\right] \tag{48}$$

Table 2 Eigenvalues and eigenvectors for closed loop longitudinal dynamics—linear regulator design

State variables	Eigenvalues $-0.690 \pm i0.682$	$-2.48 \pm i2.52$	-0.834
δu	1.0	-0.001	1.0
δw	-0.035	$0.061 \pm i0.065$	0.006
$\theta \times 10^2$	$3.47 \mp i10.2$	$-0.009 \pm i0.020$	3.76
$q \times 10^2$	$4.56 \pm i9.4$	$-0.028 \mp i0.071$	-3.14
RN	$-0.316 \pm i0.050$	1.0	0.061

Table 3 Eigenvalues and eigenvectors for closed loop lateral dynamics—linear regulator design

State variables	Eigenvalues $-0.698 \pm i0.712$	$-1.25 \pm i0.924$	-1.11
$\delta\psi \times 10^2$	$0.532 \pm i0.772$	$22.3 \mp i7.02$	-2.50
$\phi \times 10^2$	$-6.37 \pm i6.82$	$-12.0 \mp i8.42$	-11.2
δv	1.0	1.0	1.0
$r \times 10^2$	$-0.921 \mp i0.160$	-21.4 ± 29.4	2.72
$p \times 10^2$	$-0.413 \mp i9.30$	22.7 ∓ 0.518	12.4

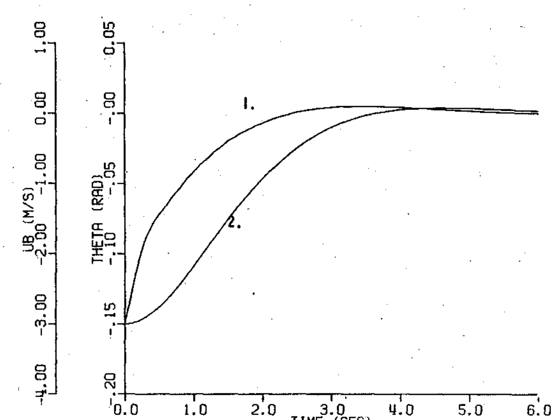

Fig. 3 Surge (1) and pitch (2) perturbation with VS control for $u_c = 3$ m/s and $\theta(0) = -0.15$ rad.

where the nominal values for the d_i are:

$$\bar{d}_1 = 7.21C_3 - 2.25C_4 + 0.875$$
$$\bar{d}_2 = 9.8C_3$$
$$\bar{d}_3 = -0.042C_3 - 0.007C_4 - 0.039$$
$$\bar{d}_4 = C_1 + 7.67C_3 - 2.46C_4 + 1.04$$
$$\bar{d}_5 = C_2 - 0.075C_4 - 0.26$$
$$\bar{d}_6 = 0.0055C_4 + 0.177 \tag{49}$$

Hence, the variable structure control that guarantees $s\dot{s} < 0$ has the form

$$\delta\text{LAT} = -\sum_{i=1}^{5} \psi_i x_i - k_s s \tag{50}$$

$$\psi_i = \begin{cases} \alpha_i, & sx_i > 0 \\ \beta_i, & sx_i < 0 \end{cases} \qquad k_s = 4.0 \tag{51}$$

In the selection of α and β, we allowed for a 100% variation in the coefficients of A and B in Eqs. (35) and (36), with the exception of $A_{3,2}$, $B_{4,2}$, and $B_{5,1}$. Then on the basis that

$$\alpha_i \geq \max\{d_i\}/\min\{d_6\}$$
$$\beta_i \leq \min\{d_i\}/\max\{d_6\} \tag{52}$$

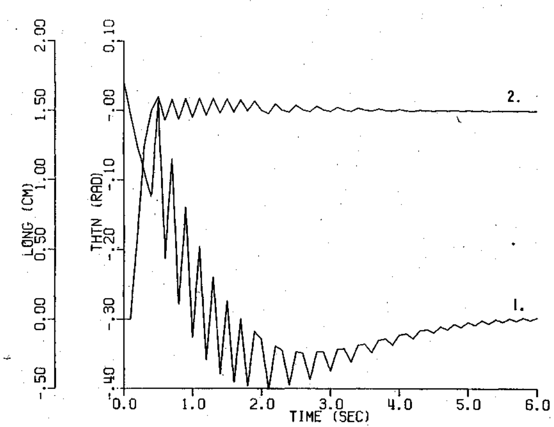

Fig. 4 Longitudinal stick (1) and nozzle angle (2) perturbation with VS control for $u_c = 3$ m/s and $\theta(0) = -0.15$ rad.

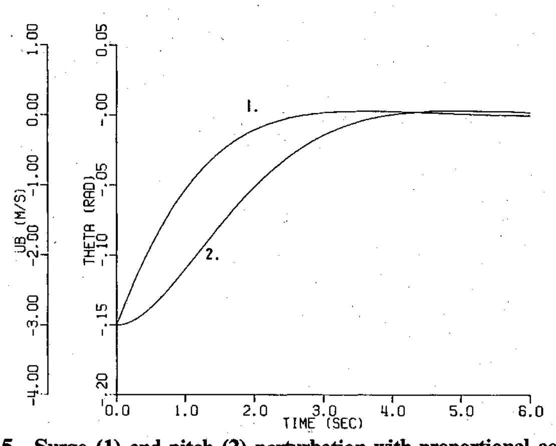

Fig. 5 Surge (1) and pitch (2) perturbation with proportional control for $u_c = 3$ m/s and $\theta(0) = -0.15$ rad.

we have the following variable structure gains:

$$\alpha = \begin{bmatrix} 4.7 \\ 5.05 \\ 0 \\ 5.56 \\ 7.88 \end{bmatrix} \qquad \beta = \begin{bmatrix} -0.245 \\ 4.96 \\ -0.50 \\ 0.304 \\ 4.585 \end{bmatrix} \tag{53}$$

The gain k_s in Eq. (51) was adjusted to yield a reasonable time to reach the sliding surface.

Linear Regulator Design

In order to have a baseline design for purposes of drawing comparisons between the VSS design and a design based on proportional state feedback, an optimal linear regulator design was carried out for both the longitudinal and lateral control. As in the VSS design, RN_c and δRUD were first replaced in terms of state variables in accordance with Eqs. (23) and (37). Then the Q and R weighing matrices were chosen such that the closed loop system dynamics approximated the VSS design in the sliding mode. The resulting longitudinal and lateral proportional feedback controls have the form:

$$u = -Kx$$

where

$$K_{\text{LONG}} = \begin{bmatrix} K_{\text{LON}} \\ K_{\text{THTN}} \end{bmatrix} = \begin{bmatrix} -0.168 & 0.149 & 14.7 & 18.8 & 0 \\ -0.056 & 0.0 & 0.475 & 0.171 & 0 \end{bmatrix} \tag{54}$$

$$K_{\text{LAT}} = [2.33 \quad 14.2 \quad 0.361 \quad 5.40 \quad 12.5] \tag{55}$$

The eigenvalues and eigenvectors for these designs are given in Tables 2 and 3. Note that the natural frequency and damping

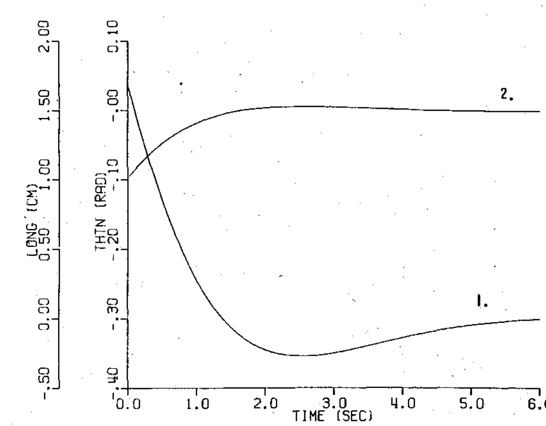

Fig. 6 Longitudinal stick (1) and nozzle angle (2) perturbation with proportional control for $u_c = 3$ m/s and $\theta(0) = -0.15$ rad.

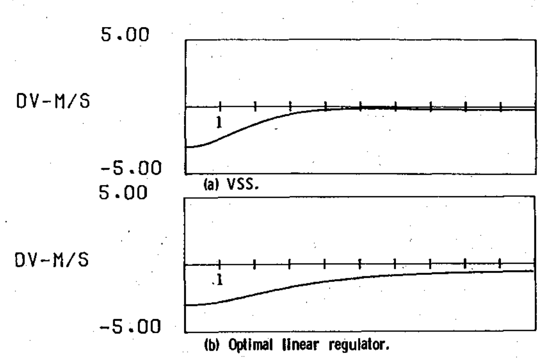

Fig. 7 Lateral velocity error time histories (10 s) for a 3-m/s lateral velocity command.

of the forward and lateral velocity modes are well matched to the corresponding sliding modes in the VSS design.

Linear Responses

The longitudinal responses for a 3 m/s command in velocity with an initial attitude error of -0.15 rad are given in Figs. 3 and 4 and 5 and 6 for the VSS and optimal linear regulator designs, respectively. For the VSS design, the sliding surface was reached in 0.4 s, and the remaining response is characteristic of an attitude only system with a natural frequency of 1.0 rad/s. Note the similarities between Figs. 3 and 5 for the surge and pitch responses and Figs. 4 and 6 for the control perturbations. Also note from Figs. 3 and 4 that despite the presence of a switching controller, the resulting velocity and attitude excursions are quite smooth.

Nonlinear Responses

This section compares the lateral performance of the VSS and the optimal linear regulator designs in the presence of kinematic and aerodynamic nonlinearities, engine and actuator dynamics, and wind disturbances.[9] First, a 10-s transient response was generated using a 3 m/s step command in lateral velocity. The aircraft was initially aligned with a constant 30 knot wind to match the flight condition used to obtain the linear design model. The resulting lateral velocity errors are shown in Fig. 7. Note that the response for the VSS is as predicted on the basis of the linear model, whereas the response for the linear control is quite sluggish. This illustrates the robustness of the VSS design. The differences in the lateral performance of the controller can be attributed to the mismatch between the linear design model and the nonlinear dynamic model.

Next, a velocity command generator was used so that the aircraft was commanded to capture and track an Approach Trajectory (AT). The AT was aligned with the freestream Wind Over Deck (WOD), at 50 deg to port, passing over the touchdown center. In this case, a ship airwake turbulence model was used. The wind disturbances were modeled from experimental data taken on a 1052 class fast frigate model,[9] and correspond to a 45 knot freestream WOD. Initial conditions were chosen such that the aircraft was 50 m from the landing point, offset laterally from the AT by 11 m and heading into the wind.

Time history responses of the lateral channel are shown in Figs. 8 and 9 for the VSS and optimal linear control designs, respectively. All responses are for a 20-s duration. In comparing the VSS and optimal linear control, the key feature is the lateral velocity error (DV) and the AT error (RG) at about 10 s into the trajectory (Figs. 8b, c and 9b, c). This corresponds to the region behind the ship where turbulence is the greatest. Note that DV and RG are significantly larger for the optimal linear control by roughly a factor of 2.5. The lateral stick shows a higher level of activity (Figs. 8d and 9d), whereas the commands are quite smooth (Figs. 8a and 9a). This illustrates the benefit of a full authority flight control system from a piloting viewpoint. The distance from the lateral switching surface is shown in Fig. 8e.

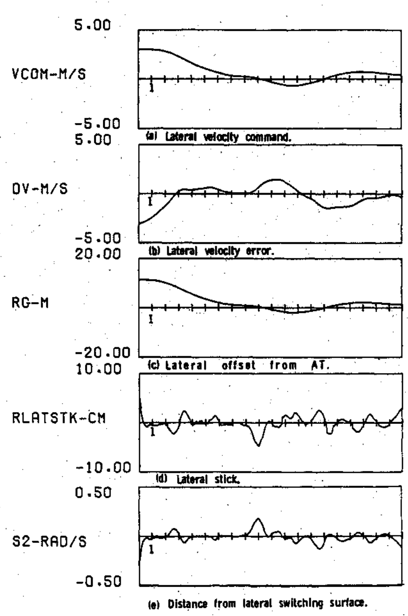

Fig. 8 VS control time histories (20 s).

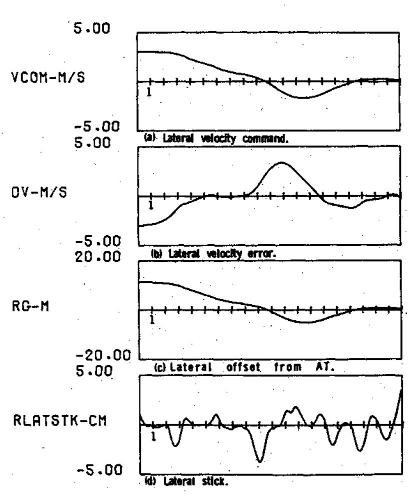

Fig. 9 Optimal linear control time histories (20 s).

IV. Conclusions

It has been shown that variable structure control theory can be used in the design of flight control systems. The resulting design is insensitive to a class of unknown but bounded parameter variations in the linearized model for the plant dynamics. Although the controller is nonlinear and contains switching elements, the resulting response (in the absence of uncertainties) is nearly identical to that obtained using linear control theory. Thus, current design specifications in terms of response settling time, overshoot, and other piloting related criteria can be used to design the variable structure control. Variable structure control in combination with a high-gain proportional component can also be used to reduce the effects of external disturbances, such as those due to wind turbulence.

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